VCO Tuning Line Noise

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A voltage-controlled oscillator (VCO) has an output signal s(t) that can be modeled by a sinusoid with peak amplitude A and instantaneous phase θ(t). The instantaneous phase is determined by the frequency tuning characteristic of the oscillator and the tuning line voltage v_c. For a VCO with a linear tuning characteristic having a zero-volt frequency fc in Hz and a tuning gain Kv in Hz/V,

$$s(t) = A\sin(\theta(t))$$

$$\theta(t) = 2\pi\int_t f(\tau)d\tau = 2\pi f_c t + 2\pi K_v \int_t v_c(\tau)d\tau$$

We determine the phase noise at the output of the oscillator with a relatively constant tuning line voltage having an rms noise density v_{mrms} (rms V/√Hz). In this case we may say f_c is the center frequency of the output signal and $v_c(t)$ is the instantaneous noise voltage on the tuning line. We can model the noise in a narrow region at an offset f_m Hz from the center frequency by a sinusoid $v_m(t)$ having peak amplitude $v_{mpk}= v_{mrms}\cdot\sqrt{2}$.

$$v_m(t) = v_{mpk}\sin(2\pi f_m t)$$

The output signal is then given by

$$s(t) = A\sin\left(2\pi f_c t + 2\pi K_v v_{mpk}\int_t \sin(2\pi f_m \tau)d\tau\right)$$

$$= A\sin\left(2\pi f_c t - \frac{K_v v_{mpk}}{f_m}\cos(2\pi f_m t)\right)$$

Applying the sum of angles and product of sines identity and making the small angle approximation (K_v v_{mpk} / f_m << 1) product we have

$$s(t) \approx A\sin\left(2\pi f_c t\right) - \frac{AK_v v_{mpk}}{2f_m}\left(\sin(2\pi(f_c - f_m)t) + \sin(2\pi(f_c + f_m)t)\right)$$

This is the narrow-band FM approximation, where a small sinusoidal modulation on the tuning line results in two small sinusoids, one on either side of the carrier at an offset equal to the modulation frequency.

The single sideband phase noise ratio (SSBN) is given by the ratio of the noise power density in one sideband to the total carrier power, usually expressed in dBc/Hz (dB below the carrier in a 1 Hz bandwidth). The total carrier power may be approximated by $A^2/2$ if the side band power is a small portion of the total power.

$$L(f_m) \approx \frac{\left(\dfrac{AK_v v_{mpk}}{2f_m}\right)^2 \Big/ 2}{A^2 / 2} = \left(\frac{K_v v_{mpk}}{2f_m}\right)^2 = \frac{1}{2}\left(\frac{K_v v_{mrms}}{f_m}\right)^2$$

where v_{mrms} is the RMS noise density, in volts rms per root Hz.

This last expression holds in general for relating noise on the tuning line, which is at baseband, to the single-sideband phase noise density at the output of an oscillator. More generally, the noise density may vary with offset frequency.

$$L(f_m) \approx \frac{1}{2}\left(\frac{K_v v_{mrms}(f_m)}{f_m}\right)^2$$

In analyzing the output phase noise spectral density of a phase locked loop, we often use the Laplace or s-domain transfer functions of the blocks in the loop, linearized about the carrier signal. For the signals in the loop that are operating with a non-baseband signal, such as the VCO, divider and reference oscillator, we can work directly with the single-sideband phase noise density and the linearized transfer functions. However, for the phase detector and loop filter we simulate or measure their noise contributions at baseband.

For these baseband noise sources that affect noise on the tuning line, their contribution to single-sideband phase noise at the loop output is given by one half their baseband noise power multiplied by the linearized transfer functions for the loop.